Intermediate Microeconomics: Topic 4 Part 1

Simon Naitram, PhD

November 6, 2020

Chapter 19: Technology

Read all of Chapter 19 (9th Edition). In the 8th Edition, this is Chapter 18.

This is the easy part of the course. If you've understood consumer theory, then firm theory is unbelievably simple and intuitive. Also, it deals with businesses rather than consumers, which is always far more fun.

What does the firm do? What do you think the firm does? What is your belief of the underlying purpose of business? Look past the razzle and dazzle (or the riff and raff) and consider what is the real single *thing* that businesses do.

Black Box In microeconomics, we will think of the firm as a black box. It's like a big machine that takes some inputs, does a little jig, and spits them out as outputs. The inputs it takes in are the *factors of production*. The outputs it spits out are finished goods.

Factors of Production What are factors of production? Factors of production are the inputs used to generate output. The normal ones you'll see in an economic model are capital and labour. You might also see 'intermediate inputs', which are the physical inputs we use (like steel, or paper). You might also see land and entrepreneurship. You can kinda see where I'm going here.

Asking what the factors needed to produce a toaster is, on the face of it, a silly question. Seems pretty simple: http://www.madehow.com/Volume-7/Toaster.html. But in fact, it is a slightly existential question. A guy called Thomas Thwaites tried to build a toaster from scratch.[1] He firstly found that it had over 400 components and sub-components. Then he found that to get the components from scratch was so difficult that it was near impossible. One really curious truth he realised was that we build very heavily on the knowledge and infrastructure built by previous societies and other firms. Quite simply, we don't live *or produce* in a vacuum. His conclusion is that "if you started absolutely from scratch, you could easily spend your life making a toaster." So, what really are the factors of production?

[1] See Thomas' Ted Talk about his toaster project here: https://www.ted.com/talks/thomas_thwaites_how_i_built_a_toaster_from_scratch?language=en&utm_campaign=tedspread&utm_medium=referral&utm_source=tedcomshare.

Bare Essentials Let's first think of the firm as its bare essentials. The firm, in its simplest form, is just that machine. For example, in the image to the right, the inputs of the machine are potatoes[2] and the outputs are potato chips. In that example, we don't consider who sources the potatoes or cleans them or puts them on the machine. We don't consider whether the potato chips that come off need to be bagged and branded, we don't consider the electricity needed to run the machine. We are simply saying that by using some number x of potatoes, we can produce some amount y of chips.

[2] Along with salt, pepper, and vinegar; but let's forget those for now.

Technology The technology available to us describes *how* we can convert potatoes into potato chips. How many perfectly sliced potato chips can we produce if the only technology we have available is a knife? How many perfectly sliced potato chips can we produce if we have a manual hand slicer? Do you think either of these could produce as many perfectly sliced potato chips as the machine I describe above?

Maximum Output We assume that the firm *knows* the most efficiency way to use technology available to produce output given a set of inputs. We can use a **production function** to describe the *maximum* amount of output that a firm can use given a specific set of inputs and a specific technology.

Production Function The function itself describes the technology, and the variables in the production function describe the inputs. In the case above, we can describe the input as x, the technology as a function $f(.)$, and the output as y. Together we state this as:

$$y = f(x), \tag{1}$$

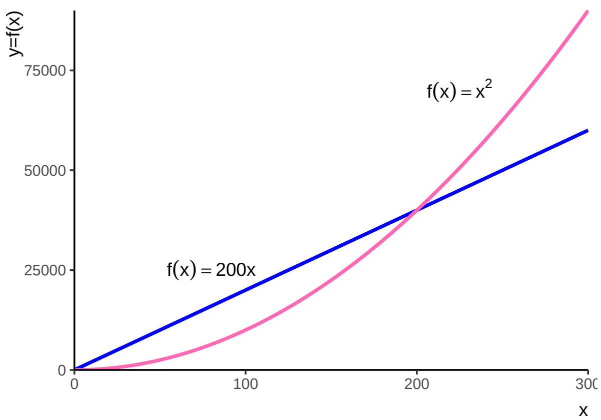
which says that we can produce a maximum amount of output y given inputs of amount x by using the technology f.

Production Set Obviously we can always produce *less* than the maximum amount given our technology and inputs. How? Well, we can just not use some of the inputs. This is the essence of the free disposal assumption. The production set is therefore all the amount of output that can be produced with a given amount of input x. It's easy to see that this runs from 0 to $y = f(x)$. This is very similar to the consumer's budget set. Why would you produce less than the maximum?

Better Technology We now have a clear way to measure whether one technology is better than the other. Which technology can produce *more* output given the same amount of inputs? For example, which is better, (1) or (2)?

$$f(x) = 200x \quad (2)$$
$$f(x) = x^2 \quad (3)$$

Trick question of course. Sometimes the first is better, sometimes the



second is better. Whoops!

Production Set We like to plot the chart of the output y against the input x. For a single input and a single output

General Extension Once you've understood the basic mechanisms, you can begin to think about how other inputs can be included in the production function. What else is needed to produce and sell potato chips? Let's say potatoes (x_1) and salt (x_2). The production function is then $y = f(x_1, x_2)$. Easy.

Cobb-Douglas Once more, let's return to our favourite functional form—the Cobb-Douglas. Be reminded that "Cobb-Douglas" isn't only used for utility functions. In fact, it started as a production function, and that is its most crucial use.[3] The standard form is:

$$f(x_1, x_2) = Ax_1^{\alpha} x_2^{\beta}. \quad (4)$$

Given the analogy with the Cobb-Douglas utility function, can you think what this might be saying? The interesting addition is parameter A, which is a 'productivity' parameter.[4] It measures the scale of production. For $y = f(x)$, if we had a single unit of x (one potato), how many chips could we get out of it? The simplest thing to do is to assume $A = 1$.

[3] The Cobb-Douglas functional form was in fact originally used by Knut Wicksell. Paul Douglas and Charles Cobb proposed that this functional form could be used to relate capital and labour to output. They tested it against data for the U.S. and so it became known as the Cobb-Douglas production function.

[4] This is an interesting way to think of technological advances. In fact, in macroeconomics, this A parameter is exactly the stock of technological advances. One of the main ways to make economies grow is by increasing this productivity parameter.

The parameters α and β are crucial. Whereas in the utility function, their *magnitude* means nothing since we can always conduct a monotonic transformation on them that scales them to sum to 1, we can't do that here. We'll see later exactly what they mean.

Properties Similar to well-behaved utility functions, we assume that production functions are **monotonic** and **convex**.

We assume they are **monotonic** because of the *free disposal* assumption: if I add a single unit of the input, it must be impossible to produce *less* than before. Why? Because I can always simply throw away that extra unit and continue producing the same amount as before. This means that having *extra* inputs should never make you produce *less*.

We assume they are **convex** because we assume that combining two *ways*[5] of producing y units should give you some way to produce at least y units again. Suppose using $(100a_1, 100a_2)$ or $(100b_1, 100b_2)$ gives us 100 units of output. Let's take a weighted average and combine these two input bundles: $(75a_1, 75a_2, 25b_1, 25b_2)$. With the a inputs, we should still be able to produce 75 units of output. With the b inputs we should still be able to produce 25 units of output. This gives a sum of 100 units of output in total still.

[5] Different ways of producing outputs are different production techniques.

Obviously convexity is a stricter assumption than monotonicity. Convexity requires that we're able to scale production up and down easily with the production technology continuing to give the same returns. In addition, we have to assume that the two production technologies (using either a's or b's) don't interfere with one another or that they're not somehow mutually exclusive.[6]

[6] You can use either method a or method b but not both.

Marginal Product The marginal product is the increase in total output y when we increase one input by one unit, holding all other inputs constant. That sounds suspiciously like a partial derivative, and it is, of course. This is a rate of change and can be expressed as:

$$\frac{\Delta y}{\Delta x_1} = \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1} \tag{5}$$

or, in derivative terms when $\Delta x_1 \to 0$ is:

$$\frac{\partial y}{\partial x_1} = \frac{\partial f(x_1, x_2)}{\partial x_1} = \text{MP}_1 \tag{6}$$

Diminishing Marginal Product Another assumption we like to make is that the marginal product of an input is decreasing as we use more of that input. That is, for $x_1 = 100$, we expect the marginal product to be lower than if $x_1 = 50$. How does this work intuitively? Well let's think of x_2 as being kitchen space; so kitchen space is fixed (it doesn't change). If you increase the number of cooks in your kitchen from 1 to 2, you'll probably get almost double the amount of food cooked. But increasing the number of cooks from 19 to 20 probably won't give you much of an increase in the amount of food cooked. Why? Well, the cooks will probably be falling over themselves, fighting over kitchen space, and shouting at one another. This is what we mean by "too many cooks spoil the both". It holds especially when we assume kitchen space to be fixed. Would this hold if we increased kitchen space?

Isoquants Can we draw a graph that shows all the different combinations of two inputs x_1 and x_2 that generates the same amount of output? Fix output at a level $\bar{y}$, and plot a graph of all the different combinations that give us exactly the same output. Similar to an indifference curve, we call this an isoquant.

Technical Rate of Subsitution Suppose we began with a level of output $\bar{y}$. Now what if we reduced input x_1 by one unit, how would we need to change x_2 get the same amount of output? Obviously if x_1 decreases, we'd need to increase the amount of x_2 if we're going to keep output the same. This gives us $\Delta x_2/\Delta x_1$, which is the technical rate of substitution. You'll notice that it's also the slope of the isoquant at $y = \bar{y}$. And we can do the same trick we did before to get:

$$\Delta y = \text{MP}_1 \Delta x_1 + \text{MP}_2 \Delta x_2 = 0 \tag{7}$$

where we say that the marginal product is the rate of change of y when we change x. To get the total change in y of a change in x, we multiply the change in x by the effect a change in x has on y. Since we need the total change in y to be zero, we set this equation equal to zero, and we can re-arrange a bit. We get:

$$\frac{\Delta x_2}{\Delta x_1} = -\frac{\text{MP}_1}{\text{MP}_2} \tag{8}$$

Obviously this can be better written in terms of partial derivatives since the marginal product is simply the partial derivative of the production function:

$$\frac{\Delta x_2}{\Delta x_1} = -\frac{\partial y/\partial x_1}{\partial y/\partial x_2} \tag{9}$$

Short Run Let's go back to the first lecture. We like to talk about the short run and the long run as if they were time periods. They're not really. We said that in the short run, it means that something is fixed. Now, more specifically, the short run is when *one or more factors of production cannot be changed*. The long run, conversely, is when we can change all factors of production. These aren't time periods, but just assumptions about whether something can be changed or not. In the long run we're all dead anyway.[7]

[7] From John Maynard Keynes: *But this long run is a misleading guide to current affairs. In the long run we are all dead.*

Fixed Variables When we want to talk about a variable being fixed, we like to put a bar over it. For example, if x_2 can't be changed in the short run, we say that it's $\bar{x}_2$. This isn't the same as saying it's exogenous. When we say it's exogenous, we consider we can't change it at all in the model. When we say it's fixed, we consider that just for the moment, we can't change it.

Returns to Scale! This, in my mind, is a really important issue. Instead of increasing a *single* factor of production,

What happens when we increase *all* inputs by the same factor? For example, if we double inputs. Does output double exactly? Does it less than double? Does it more than double? The answer depends entirely on our production technology. This is easy to show in a Cobb-Douglas production function: $f(x_1, x_2) = x_1^a x_2^b$.

- **When output exactly doubles.** We say that there are constant returns to scale. This happens when $a + b = 1$.
- **When output less than doubles.** We say that there are decreasing returns to scale. This happens when $a + b < 1$.
- **When output more than doubles.** We say that there are increasing returns to scale. This happens when $a + b > 1$.

Are all of these plausible? Well if we are able to double *all* factors of production, there should never be a case where doubling all factors leads to less than double output. We can literally reproduce everything, and get exactly the same result. As in the kitchen example, the only reason doubling inputs shouldn't result in output doubling is when *something* is being held fixed. But if we're doubling all inputs, nothing should be held fixed. Well, the problem is, given our discussion on what are actually factors of production, there are some very real factors of production that we might be missing or that might be unobservable. For example, if the government is an input into the production process, can we easily double the government? Or, another example, how easy is it to forget to include entrepreneurial effort in the production function?

Miles Kimball One economics professor, Miles Kimball, says there's no such thing as decreasing returns to scale. From his blog:[8]

[8] https://blog.supplysideliberal.com/post/2017/5/29/there-is-no-such-thing-as-decreasing-returns-to-s

> There is no such thing as decreasing returns to scale. Everything people are tempted to call decreasing returns to scale has a more accurate

name. The reason there is no such thing as decreasing returns to scale was explained well by Tjalling Koopmans in his 1957 book Three Essays on the State of Economic Science. The argument is the replication argument: if all factors are duplicated, then an identical copy of the production process can be set up and output will be doubled. It might be possible to do better than simply duplicating the original production process, but there is no reason to do worse. In any case, doing worse is better described as stupidity, using an inappropriate organizational structure, or X-inefficiency rather than as decreasing returns to scale.

The replication argument is extremely valuable as a guide to thinking about production since it often identifies a factor of production that might otherwise be ignored, such as land, managerial time, or entrepreneurial oversight. Thinking through what difficulties there might be in practice in duplicating a production process at the same efficiency helps to identify key factors of production if those factors of production were not already obvious.

So even though we *observe* decreasing returns to scale in reality, what is actually happening is that we are missing some of the factors of production. So maybe we are observing

$$f(x_1, x_2, x_3) = x_1^a x_2^b x_3^c, \qquad a + b + c < 1 \tag{10}$$

when in reality, we are simply missing a factor of production, and the real production function is:

$$f(x_1, x_2, x_3, x_4) = x_1^a x_2^b x_3^c x_4^d, \qquad a + b + c + d = 1. \tag{11}$$