Intermediate Microeconomics: Revealed Preference and Slutsky Equation

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Chapter 6: Revealed Preference

Read Sections 7.1 to 7.4, and 7.6. Make sure you understand the intuition behind revealed preferences, and make sure you know what WARP and SARP are.

The truth is that its nearly impossible to actually know what a person's utility function looks like. We'd have to sit there and ask them all their preferences over all sorts of quantities of things. But, funny thing about life is that people's behaviour reveals their beliefs. More pertinently, people's choices reveal their preferences. This chapter is about using people's actual choices to figure out what it is they prefer. It's the simple notion of watching what a person does rather than listening to what they say.

This idea that we can reveal people's preferences depends on what we assume about their behaviour. Remember when we said that we could rationalise any decision as reflecting a person's preferences and/or information? Well this is us observing people's preferences and rationalising them. In order to rationalise them, we need a theory about how people behave. This is *consumer theory*, the theory of how consumers choose optimal consumption bundles, which we've been building for the entire semester.

Direct Revealed Preference The main assumption we will make, then, is that people are utility-maximising. That is, given what bundles are available to them, they choose the one they prefer the most. This is simple but powerful. Why? Imagine a person with income m chooses (purchases) a bundle (x_1, x_2) at prices p_1 and p_2.

- This means that of all the possible bundles they could buy with their income, they chose bundle (x_1, x_2). Think of (x_1, x_2) then as 'the chosen one'.
- Consider another bundle (y_1, y_2). If (y_1, y_2) is *affordable* to the consumer, that is, if $p_1 y_1 + p_2 y_2 \leq m$, then this option was available to the consumer when they chose (x_1, x_2). This obviously means that the consumer simply preferred (x_1, x_2) to (y_1, y_2)!
- We say that (x_1, x_2) is **directly revealed preferred** to (y_1, y_2). That

is, the consumer's choices have revealed to us what their preferences over these two bundles are.

- Importantly then, we can say that $(x_1, x_2) \succ (y_1, y_2)$.

All we're saying here is that the consumer could've chosen *either* bundle (x_1, x_2) or bundle (y_1, y_2). The fact that they chose (x_1, x_2) when (y_1, y_2) was available tells us a very clear message about their preferences.

Make sure you understand the principle of revealed preference from the Varian.

Indirect Revealed Preference Here's where transitivity comes in super-useful. We'll combine transitivity—the assumption that we can make logical statements about a person's preferences—with direct revealed preference. *Imagine that prices change from p_1, p_2 to q_1, q_2.*

- Suppose that at those prices, the consumer's original preference (x_1, x_2) is no longer affordable, and therefore cannot be chosen.
- Suppose at these new prices, the bundle (y_1, y_2) is the consumer's optimal choice.
- Now suppose at these new prices q_1, q_2, there's another bundle (z_1, z_2) which is affordable, but is not chosen. As before, this implies that (y_1, y_2) is directly revealed preferred to (z_1, z_2).
- If, from before, we have that (x_1, x_2) is directly revealed preferred to (y_1, y_2), and now we have that (y_1, y_2) is directly revealed preferred to (z_1, z_2), then if the consumer's preferences are transitive, then we can say that (x_1, x_2) is **indirectly revealed preferred** to (z_1, z_2).

This chain of logical inference can go on for as long as we want.

Weak Axiom of Revealed Preference If the theory of consumer behaviour we've built holds true, then a consumer who chose (x_1, x_2) when (y_1, y_2) was available, should not turn around and choose (y_1, y_2) when (x_1, x_2) is available. This is the weak axiom of revealed preference. We're supposing that consumer decisions should be logically coherent. If a consumer does this, then it must be that something underlying the environment has changed—that their preferences have changed or some unobserved condition has changed.

Strong Axiom of Revealed Preference The strong axiom is simply an extension of the weak axiom to indirect revealed preference. If bundle (x_1, x_2) was indirectly revealed preferred to bundle (z_1, z_2), then bundle (z_1, z_2) should not be indirectly revealed preferred to bundle (x_1, x_2).

Slutsky Equation

Read Sections 8.0 to 8.4. Ensure that you understand and can explain what the income and substitution effects are, and how they matter for total changes in demand.

When the price of a good changes, what happens? Well, we know from our previous lectures on demand that we can use comparative statics to tell us what the change in demand for that good will be. But in this section, we want to break that whole change in demand into two parts. In this lecture, we're dissecting the law of demand.

The Substitution Effect The first thing that happens when price changes is the substitution effect. Remember that the *slope* of the budget line is given as the ratio of the two prices: $-p_1/p_2$. Remember that at the optimal, the marginal rate of substitution is exactly equal to this ratio. So when one of these prices changes, the ratio must change. That is, the slope of the budget line must change. Therefore the marginal rate of substitution at the optimal must change. And remember why the marginal rate of substitution changes along the indifference curve? Because we prefer averages to extremes, so when we have more of a good we're happy to *give up* more of it. That is, the change in the ratio of prices induces an optimal **substitution effect.** The more basic idea is that (holding all else constant), when the ratio of prices between two goods changes, we'll tend to want less of the more expensive good and more of the good that is now cheaper.

The Income Effect The second thing that happens when a price changes is that you immediately become richer or poorer. If a price increases, you're poorer. Poorer in the sense that you can now afford less things. Conversely, if a price falls, you're richer since you can afford more things. As we saw before, when prices fall, your indifference curve shifts outwards since you can afford more (and better) bundles.

The Process How do we go about dissecting these effects? Well in order to figure out the substitution effect, we have to hold the income effect constant. And vice versa, obviously. The income effect happens when your *real purchasing power changes* as a result of the price change. Therefore, to figure out what the substitution effect is, we need the examine the impact of a price change *holding real purchasing power constant.*

Holding Purchasing Power Constant This means that we need make sure that the consumer can *only* buy as much as they were buying before. This means, we need to change the price ratio (change the slope of the budget curve), but also change the consumer's income so that there is no change in their purchasing power.

Suppose before the price change, the consumer was purchasing (x_1, x_2) of the goods. With original prices p_1 and p_2, it means they were spending $p_1 x_1 + p_2 x_2 = m$ dollars. Now if the price of good 1 changes from p_1 to p_1', how do we ensure that the consumer can *only just* afford the bundle they were buying before? Well, we need to alter their income (either increase or decrease depending on the direction of the price change). Notice that if the price fell, then consumer would now (in reality) have more money to spend if they kept purchasing their old bundle so that $p_1' x_1 + p_2 x_2 < m$. So we construct a fictional amount of income m' which makes this expression equal again, such that:

$$p_1' x_1 + p_2 x_2 = m'.$$

This gives us a new budget line Notice now it has a new slope $-p_1'/p_2$, but both the original and the new (imaginary) budget line can *only just* afford to purchase the original optimal bundle (x_1, x_2). This is the idea of **holding purchasing power constant**. The consumer can only just afford the bundle they were buying before.

Now, given that we've worked out this imaginary budget line which holds purchasing power constant, even though the consumer can afford the bundle they were purchasing before, will they continue to buy it? Well, to answer that, we simply solve the optimal choice problem again, using the new budget constraint that we've created. We will get a new optimal bundle: (x_1^p, x_2^p).

The substitution effect is therefore the change in the optimal demand for good 1, from x_1 to x_1^I. Or more explicitly, the change in the optimal demand using the demand function:

$$\Delta x_1^S = x_1^*(p_1, m) - x_1^*(p_1', m').$$

We'll call this substitution effect Δx_1^S.

Holding the Price Ratio Constant Conversely, to find the income effect, we need to hold the price ratio constant, and allow the consumer's purchasing power to change. How do we do this? Well, we've already created an imaginary budget constraint where only the price ratio has changed. What we're interested in now is, holding the price ratio constant at $-p_1', p_2$, what would be the effect on demand if we allow income to change from our imaginary income m' to the actual income m. This is now allowing the consumer's purchasing power to change. This is the second step of the process. This means we care about the difference between the optimal choice at the imaginary budget constraint

$$p_1'x_1 + p_2x_2 = m'$$

and the new actual optimal choice at the actual new budget constraint

$$p_1'x_1 + p_2x_2 = m.$$

Once again, if we work through the optimal choice problem for both budget constraints, we get the shift in demand for good 1 from x_1^p to a new demand x_1'. Or, plugging it into our demand function, we get that the income effect is the difference

$$\Delta x_1^I = x_1(p_1', m') - x_1(p_1', m).$$

That is, we keep the price ratio constant, but allow the consumer's purchasing power to change from m', which we fabricated, to m, which is the consumer's actual income (which hasn't actually changed).

Total Change Notice then that all we've done is to break the total change in demand resulting from a change in p_1 into two steps:

1. Substitution effect: hold purchasing power constant by changing m and allowing price p_1 to change

2. Income effect: using the new price ratio, allow purchasing power to change from the constant level we created above.

This implies that the total change in demand is simply the sum of these two effects. [1] That is:

[1] Remember, just plugging in the different prices or incomes into the demand function gives us the new optimal demand.

$$\Delta x_1 = \Delta x_1^S + \Delta x_1^I$$

or more quantitatively:

$$\Delta x_1 = x_1^*(p_1, m) - x_1^*(p_1', m') + x_1(p_1', m') - x_1(p_1', m) \quad (1)$$
$$\Delta x_1 = x_1^*(p_1, m) - x_1(p_1', m) \quad (2)$$
$$(3)$$

which is the exact definition of Δx_1.

The Sign of the Substitution Effect Have a look at the substitution effect in the context of revealed preference. By construction, we create a second budget line that passes through our original optimal bundle. This means the optimal bundle is still affordable. According to the idea of revealed preference, if a consumer is optimising, then when the budget line changes, they won't choose anything that was affordable before. Try to use a standard revealed preference argument to understand why the substitution effect is always negative. That is, an increase in the price of good 1 always leads the consumer to substitute away from good 1. Draw this graphically and see if you can understand it.

Check the graphical interpretation of the substitution and income effects in the text. Make sure you understand it both graphically and mathematically.